Inspiring Greatness

ISD

INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

Probability and Statistics

April 22, 2020

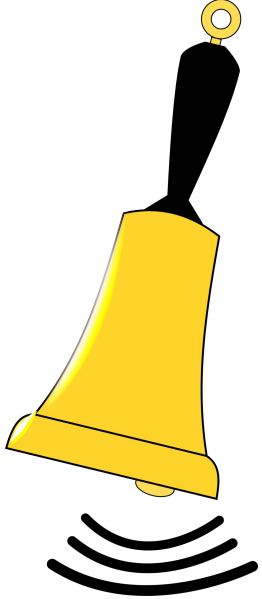
Probability and Statistics
Lesson: April 22, 2020

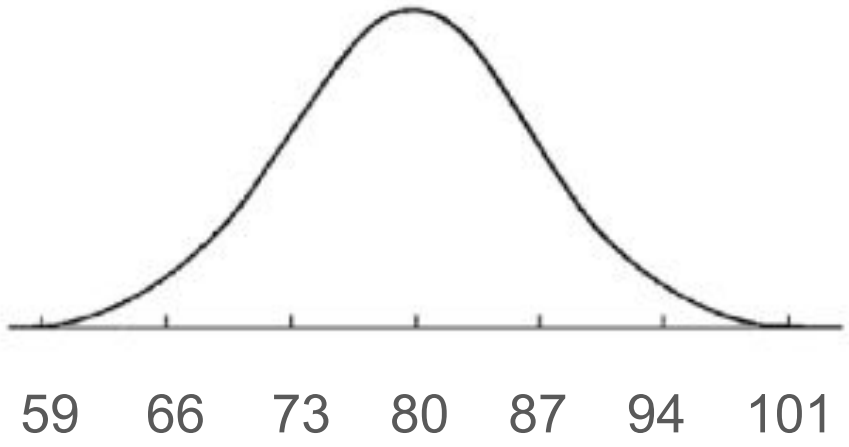
Objective/Learning Target:

Students will be able to calculate the z-score of a set of data

Let's Get Started!

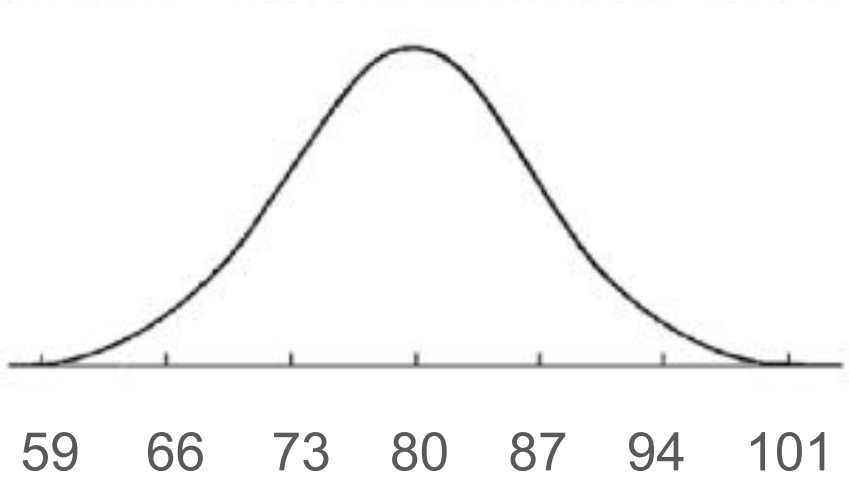
Carlos and Mary were working on drawing some conclusions about the success of their students on a recent test. They mapped out this curve based on finding that the average test score was an 80% and the standard deviation was 7.

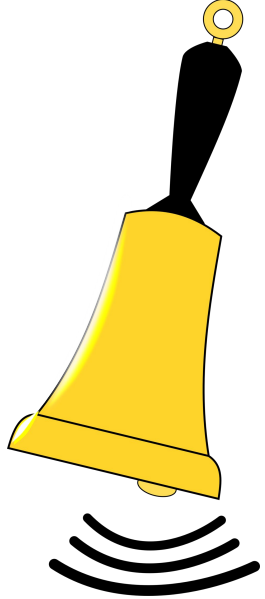




Let's Get Started!

Carlos made the claim that 68% of the students scored at least a 73% on the test. Mary claims that it is actually higher; it is really 84% Who is correct?





Let's Get Started! ANSWER

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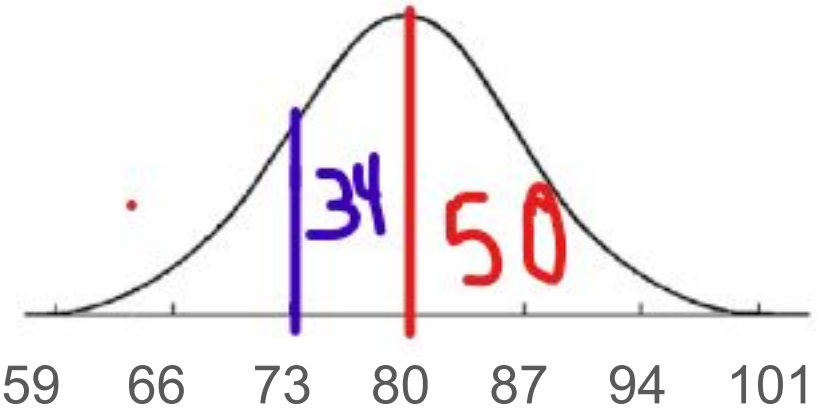
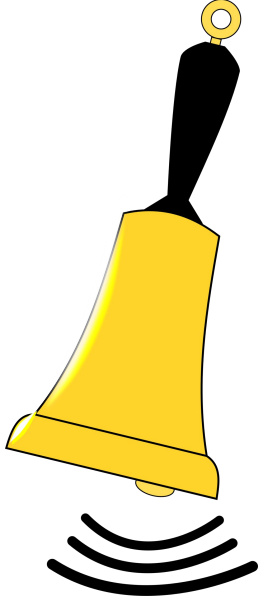


Mary is correct. 50% of the students scored 80 or above. Since 73 is one standard deviation away from the mean, that means that another 34% scored between 73 and 80.

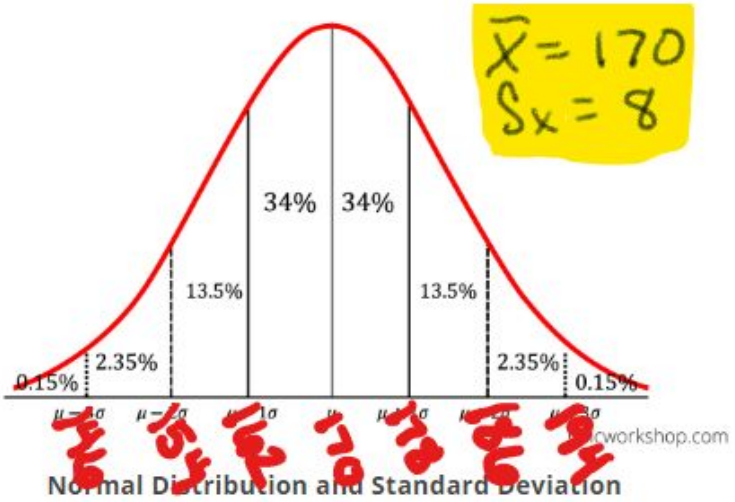
This is a total of 84%



Z-Scores

We have been practicing with percentages when the number we are evaluating falls perfectly on a standard deviation. But what happens when it doesn't?

In the example on the right, you know what % is between 154 and 178 because 154 and 178 fall perfectly on a standard deviation. You can use the Empirical Rule to add the standard percentages to get an answer of 13.5+34+34 = 81.5%

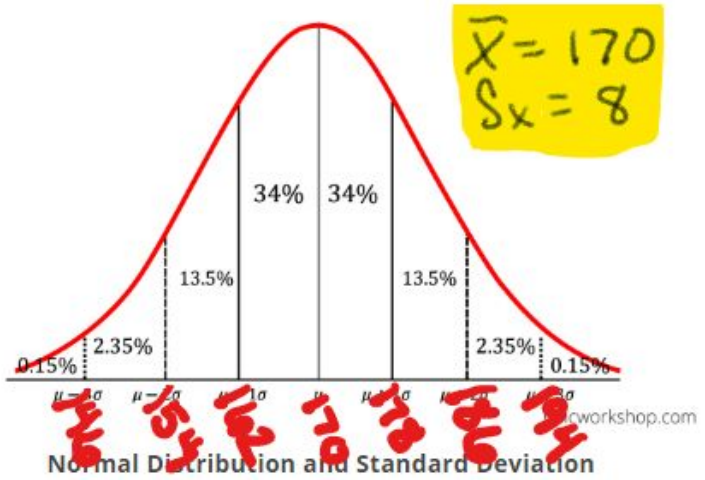


Z-Scores

But what happens if you need the percentage that falls between 154 and 172? Or the percentage that is below 180?

154 is on a perfect standard deviation, but 172 is not and neither is 180.

This is where we introduce the Z-Score formula to help you get an exact placement on the graph so that you can then use a chart tomorrow to find the exact percentages you need.



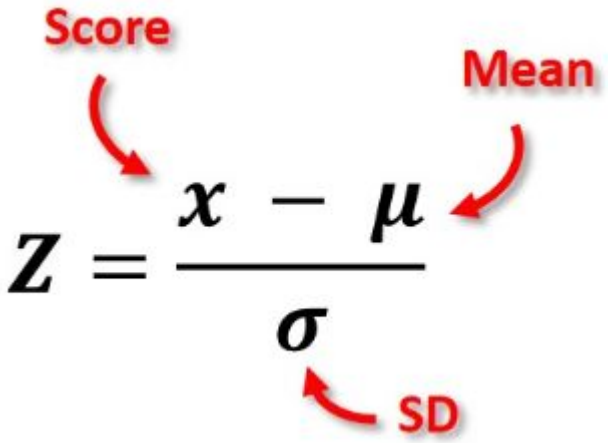
Z-Score Formula

Today we will focus only on the formula and how to calculate it. (Tomorrow we will work on connecting it to the percentage chart that goes with it.)

In order to use the Z-Score Formula you will need 3 pieces of information:

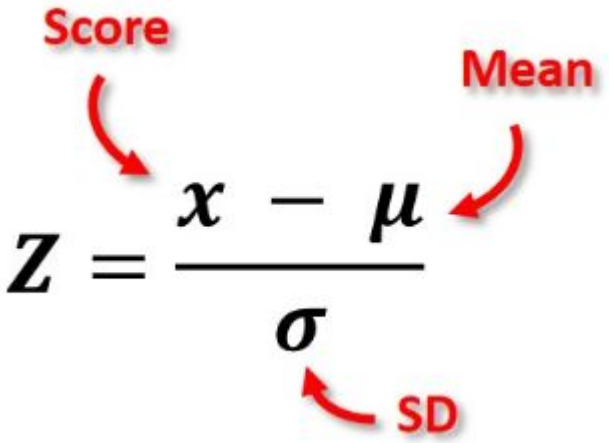
1. The number you are looking for (x)
2. The Mean
3. The Standard Deviation

The Z-Score is going to tell you EXACTLY (in decimal form) how many standard deviations you are from the mean.



From TopTipBio.com

Z-Scores



***** Be sure that when you subtract on top, ALWAYS do your # minus the Mean!!** *(if you subtract it backwards you will get a wrong answer)*

In our example on slide 5 (Mean: 170 & SD: 8) we need to calculate the Z-Score for:

1. 154: $\frac{154 - 170}{8} = \frac{-16}{8} = -2$

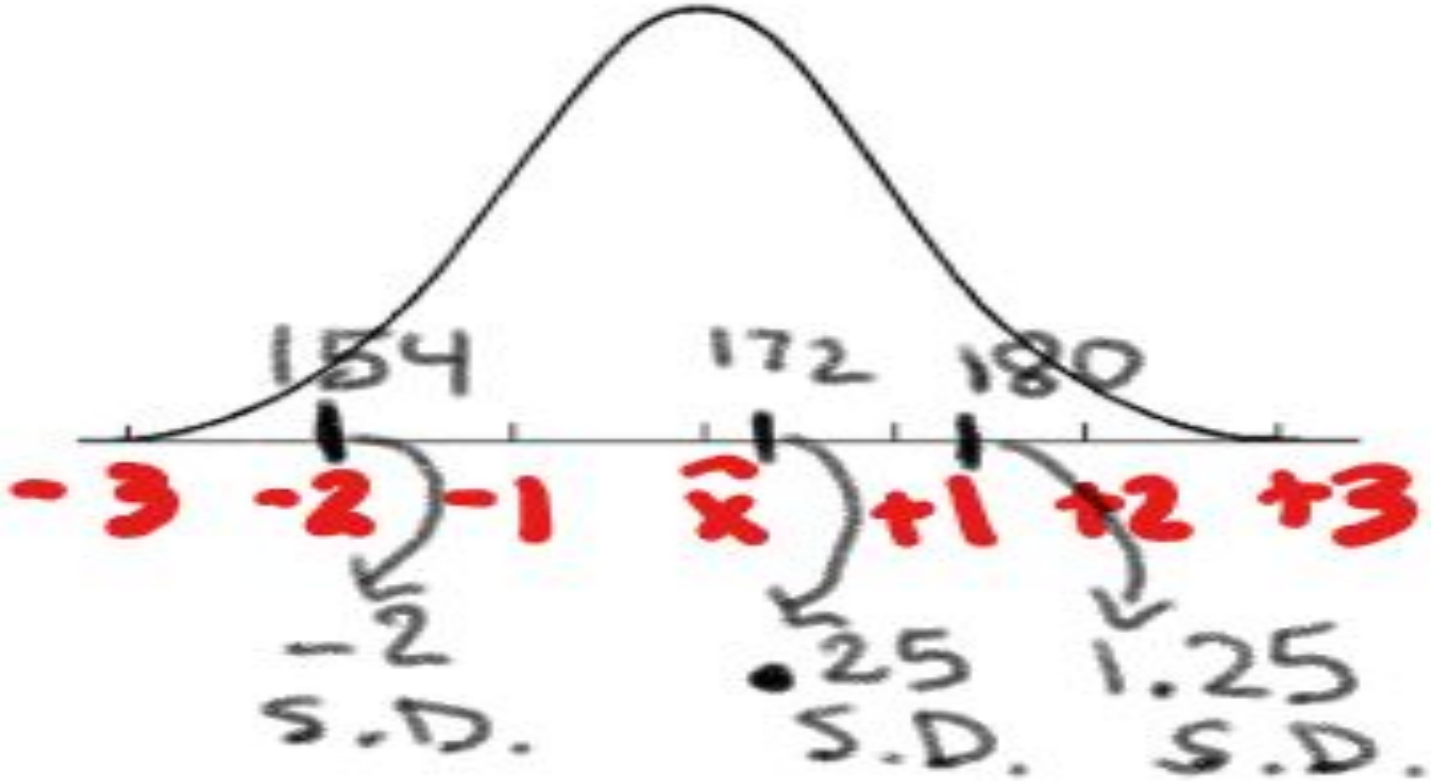
This answer should make sense because 154 is sitting at -2 (2 to the left) standard deviations from the mean on our bell curve graph.

2. 172: $\frac{172 - 170}{8} = \frac{2}{8} = .25$

This answer means that 172 falls .25 standard deviations to the right of 170 because it is positive .25 (¼ of the way between 170 & 178). This should make sense because 172 is closer to 170.

3. 180: $\frac{180 - 170}{8} = \frac{10}{8} = 1.25$

This answer means that 180 falls 1.25 standard deviations to the right of 170 because it is positive 1.25. If it had been negative it would fall to the left of the mean.



The Z-Score is going to tell you (in decimal form) where to put the tick mark on your graph

Video

Watch this video to see more examples:

Z-Score Demo 1
(only watch section 3:10 to 4:30 of this video)

Z-Score Demo 2

Working Backwards:

Sometimes you will be given the Z-Score and be asked to find the X value that corresponds to it.

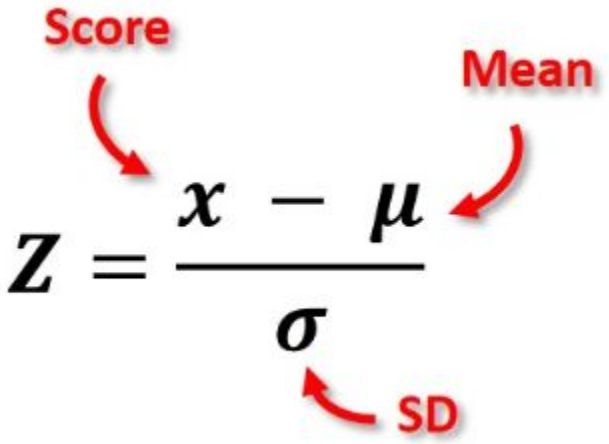
It is basically an Algebra Problem and you are solving for X

This short video will walk you through it:

Z-Score Working Backwards (finding X)

Z-Scores and X-Values

*** Keep in mind that you will be asked to find both Z-Scores AND X values so be sure that you can work the formula forwards AND backwards.



Practice Example #1

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. Suppose they want to know how many students scored a C (70%) or higher. Find the z-score of 70% and describe what this number means.

Practice Example #1 ANSWER

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. Suppose they want to know how many students scored a C (70%) or higher. Find the z-score of 70% and describe what this number means.

$$Z = \frac{70 - 80}{7}$$

= -1.43 This means that 70 is 1.43 standard deviations to the left of the mean (because it's negative)

Practice Example #2

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. What test score would have a z-score of 2.5?

Practice Example #2 ANSWER

In our bell ringer, Carlos and Mary were looking at test scores of their students. They found the mean to be 80% with a standard deviation of 7%. What test score would have a z-score of 2.5?

$$2.5 = \frac{x - 80}{7}$$

-----> 17.5 = x - 80 (multiply both sides by 7)

-----> x = 97.5 (add 80 to both sides)